

Assignment 10

csci2200, Algorithms

Instructions:

- HONOR CODE: WORK ON THIS ASSIGNMENT WITH AT MOST ONE PARTNER. BETWEEN DIFFERENT TEAMS, COLLABORATION IS AT LEVEL 1 [VERBAL COLLABORATION ONLY]
 - Write each problem on a separate page; If a problem has multiple parts, you can write all parts on the same page, as long as you leave space in between them.
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1. **Finding the singleton:** You are given a sorted array of numbers where every value except one appears exactly twice, and one value appears only once. Design an efficient algorithm for finding which value appears only once.

Note: A general solution should not assume anything about the numbers in the array; specifically, they may not be in a small range, and may not be consecutive.

Example: Here are some example inputs to the problem:

1, 1, 2, 2, 3, 4, 4, 5, 5, 6, 6, 7, 7, 8, 8

10, 10, 17, 17, 18, 18, 19, 19, 21, 21, 23

1, 3, 3, 5, 5, 7, 7, 8, 8, 9, 9, 10, 10

We expect: pseudocode, a clear English description of what the algorithm is doing and why it is correct; running time analysis.

2. **Party problem:** Suppose you are in charge of planning a party for an institution (for e.g., Bowdoin College). The institution has a hierarchical structure, which forms a tree rooted at the President. Linked directly under the president are the people who report directly to the

president, and their “children” are the people who report directly to them, and so on. Part of this hierarchy are the faculty, grouped by department; all faculty in the same department have the department chair as their parent. Each faculty has as children all the student advisees. Assume that every person is listed at the highest possible position in the tree and there are no double affiliations: everybody has one and only one supervisor in this hierarchy and every student has only one advisor.

You have access to a secret database which ranks each faculty/staff/student with a conviviality rating (a real number, which can be negative if the person is really grumpy or boring). As the party organizer, your goal is to maximize the fun, so you decide to select the guest list such that: for any guest, his/her immediate supervisor is not also a guest. Note that this means that the President may not be invited, but it is what it is. You’re doing your job.

You are given a tree that describes the structure of the institution. Each node has a (down) pointer to its left-most child, and a (right) pointer to its next sibling. Each node also holds a name and a conviviality ranking. Describe an algorithm to make up a guest list that maximizes the sum of the conviviality rankings of the guests. Analyze the running time of your algorithm. Assume the size of the tree (number of nodes) is n .

Hint: Aim for $O(n)$ time.

[We expect: Explanation of your choice of subproblem and its parameters (i.e. what function you will compute, what it returns and what are its parameters); justification of optimal substructure; pseudocode and an English description of your algorithm, as well as analysis of its running time.]